A spherically symmetric and stationary universe from a weak modification of general relativity

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Abstract

It is shown that a weak modification of general relativity, in the linearized approach, renders a spherically symmetric and stationary model of the universe. This is due to the presence of a third mode of polarization in the linearized gravity in which a "curvature" energy term is present. Such an energy can, in principle, be identified as the Dark Energy. The model can also help to a better understanding of the framework of the Einstein-Vlasov system.

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The accelerated expansion of the Universe that is currently purported from observations of SNe Ia suggests that cosmological dynamics is dominated by a "new" substance of the universe constituents dubbed as Dark Energy, which is able to provide a large negative pressure to account for the late-time accelerate expansion. This is the standard picture, in which such a new ingredient is considered as a source of the *right-hand-side* of the field equations. It is posed that it should be some form of un-clustered non-zero vacuum energy which, together with the clustered Dark Matter, drives the global dynamics. This is the so-called "concordance model"(ΛCDM) which gives, in agreement with the data analysis of the observations of the Cosmic Microwave Background Radiation (CMBR), Lyman Limit Systems (LLS) and type la supernovae (SNe Ia), a good framework for understanding the currently observed Universe. However, the ΛCDM presents several shortcomings as the well known"coincidence" and "cosmological constant" problems [1].

An alternative approach to explain the purported late-time acceleration of the universe is to change the *left hand side* of the field equations, and to inquire whether the observed cosmic dynamics can be achieved by extending general relativity [2, 3, 4]. In this different context, it is not required to search candidates for Dark Energy and Dark Matter, which until to date, have not been found, but rather it claims that only the "observed" ingredients: curvature and baryon matter, have to be taken into account. Considering this point of view, one can posit that gravity is not scale-invariant [5]. In so doing, one allows for a room for alternative theories to be opened [6, 7, 8]. In principle, interesting Dark Energy and Dark Matter models can be built by considering $f(R)$ theories of gravity [5, 9] (here R is the Ricci curvature scalar).

In this perspective, even the sensitive detectors of gravitational waves like bars and interferometers (i.e., those which are currently in operation and the ones which are in a phase of planning and proposal stages[10, 11], could, in principle, test the physical consistency of general relativity or of any other theory of gravitation. This is because in the context of Extended Theories of Gravity important differences with respect to general relativity show up after studying the linearized theory[12, 13, 14, 15].

In this letter is shown that a weak modification of general relativity inspired by f(R) theories of gravity leads to a spherically symmetric and stationary model of the universe. (The analysis is performed in the context of the linearized theory). Such a feature appears due to the presence of a third mode of polarization in the linearized gravity in which a "curvature" energy is present. This peculiar behavior may in some respect resemble the curvaton field dynamics. That is, it does not itself drive inflation; it merely generates curvature perturbations (which could be stationary) at late times after the inflaton field has decayed and the decay products have redshifted away, when the curvaton is the dominant component of the energy density. We recall that the curvaton field is a light scalar field during inflation whose quantum fluctuations produce the primordial density perturbations in a scenario for the origin of structure formation. It is posed that spatial variations in the curvaton density are then transferred to the cosmic background radiation density when the curvaton decays some time after inflation. See the following short list of references [16].

Such an energy can, in principle, be identified as the Dark Energy. The model can also help to have a better understanding of the Einstein-Vlasov system [17, 18, 19, 20].

Let us consider the action

$$S = \int d^4x\sqrt{-g}f_0 R^{1+\varepsilon} + \mathcal{L}_m \tag{1}$$

Equation (1) is a particular choice in $f(R)$ theories of gravity [2, 3, 4, 5, 6, 7, 9, 13].

In cosmology the action (1) has been analysed in [24] in a rather different cosmological scenario as compared to the one analysed in this letter. In the limit $\varepsilon \to 0$ and $f_0 = 1$ it recovers the canonical form of the Einstein-Hilbert action of general relativity [21, 22], i.e.

$$S=\int d^4x\sqrt{-g}R+\mathcal{L}_m \tag{2}$$

Criticisms on $f(R)$ theories of gravity arises from the fact that lots of such theories can be excluded by requirements of Cosmology and Solar System tests [23]. However, in the case of the action (1), the discrepancy with respect to the standard General Relativity is very weak, because ε is a very small real parameter. Thus, the mentioned constraints could, in principle, be satisfied. In particular the authors of [23] found

$$0\leq\varepsilon\leq7.2*10^{-19}. \tag{3}$$

We have also to emphasize that fundamental constrains can be renormalized in order to obtain $f_0=1$.

Because we want to study interactions at cosmological scales, the linearized theory in vacuum, i.e. with $\mathcal{L}_m=0$, must be considered. Notice that it gives a better approximation than the Newtonian theory [28] and the importance of the linearized theory in a cosmological framework has also been recently emphasized by George Ellis [29]. Therefore, we will analyse the pure curvature action

$$S=\int d^4x\sqrt{-g}f_0R^{1+\varepsilon}. \tag{4}$$

Also notice that the theory arising from such an action has been recently linearized in [30], but a review is needed for a better understanding of the theoretical framework.

By varying the action (4) with respect to $g_{\mu\nu}$, the field equations are obtained (through this paper the convention $G=1$, $c=1$ and $\hbar=1$ will be used) [12, 13, 30]

$$G_{\mu\nu}=\frac{1}{(1+\varepsilon)f_0R^{\varepsilon}}\{-\frac{1}{2}g_{\mu\nu}\varepsilon f_0R^{1+\varepsilon}+[(1+\varepsilon)f_0R^{\varepsilon}]_{;\mu;\nu}-g_{\mu\nu}\Box[(1+\varepsilon)f_0R^{\varepsilon}]\}. \tag{5}$$

By taking the trace of the field equations (5) one gets

$$\Box(1+\varepsilon)f_0R^{\varepsilon}=\frac{(1-\varepsilon)}{3}f_0R^{1+\varepsilon}. \tag{6}$$

Then, by making the identifications [25, 30]

$$\Phi\rightarrow(1+\varepsilon)f_0R^{\varepsilon}\qquad\text{and}\qquad\tfrac{dV}{d\Phi}\rightarrow\tfrac{(1-\varepsilon)}{3}f_0R^{1+\varepsilon} \tag{7}$$

a Klein - Gordon equation for the effective Φ scalar field is obtained. It can be written as

$$\Box\Phi=\frac{dV}{d\Phi}. \tag{8}$$

To study gravitational waves, the linearized theory has to be analyzed, with a little perturbation of the background, which is assumed given by a near

Minkowskian background, i.e. a Minkowskian background plus $\Phi = \Phi_0$ (the Ricci scalar is assumed constant in the background) [13, 25, 30]. We also assume Φ_0 to be a minimum for the effective potential V:

$$V \simeq \frac{1}{2}\alpha\delta\Phi^2 \Rightarrow \frac{dV}{d\Phi} \simeq m^2\delta\Phi, \tag{9}$$

and the constant m has mass dimension.

Putting

$$\begin{gathered} g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu} \\ \Phi = \Phi_0 + \delta\Phi. \end{gathered} \tag{10}$$

to first order in $h_{\mu\nu}$ and $\delta\Phi$, calling $\widetilde{R}_{\mu\nu\rho\sigma}$, $\widetilde{R}_{\mu\nu}$ and $\widetilde{R}$ the linearized quantity which correspond to $R_{\mu\nu\rho\sigma}$, $R_{\mu\nu}$ and R, the linearized field equations are obtained [13, 25, 30]:

$$\begin{gathered} \widetilde{R}_{\mu\nu} - \frac{\widetilde{R}}{2}\eta_{\mu\nu} = (\partial_\mu\partial_\nu h_m - \eta_{\mu\nu}\Box h_m) \\ \Box h_m = m^2 h_m, \end{gathered} \tag{11}$$

where

$$h_m \equiv \frac{\delta\Phi}{\Phi_0}. \tag{12}$$

Then, from the second of eqs. (11), one can define the mass like [13, 25, 30]

$$m \equiv \sqrt{\frac{\Box h_m}{h_m}} = \sqrt{\frac{\Box\delta\Phi}{\delta\Phi}} = \sqrt{\frac{\Box\delta R^\varepsilon}{\delta R^\varepsilon}}. \tag{13}$$

Thus, as the mass is generated by variation of the Ricci scalar, we can say that, in a certain sense, it is generated by variation of spacetime curvature, re-obtaining the same result of [13, 25, 30].

Note that, in the present case, the theory is suitable as the modification of General Relativity is very weak and in agreement with requirements of Cosmology and Solar System tests [23].

$\widetilde{R}_{\mu\nu\rho\sigma}$ and eqs. (11) are invariants under gauge transformations of the type [12, 13, 30]

$$\begin{gathered} h_{\mu\nu} \to h'_{\mu\nu} = h_{\mu\nu} - \partial_{(\mu}\epsilon_{\nu)} \\ h_\varepsilon \to h'_\varepsilon = h_\varepsilon. \end{gathered} \tag{14}$$

Therefore, one can define [30]

$$\bar{h}_{\mu\nu} \equiv h_{\mu\nu} - \frac{h}{2}\eta_{\mu\nu} + \eta_{\mu\nu}h_\varepsilon \tag{15}$$

and, considering the transform for the parameter ϵ^μ as

$$\Box\epsilon_\nu = \partial^\mu \bar{h}_{\mu\nu}. \tag{16}$$

On this basis one can choose a gauge similar to the Lorentz gauge that is used when studying electromagnetic waves. It reads: [30]

$$\partial^\mu \bar{h}_{\mu\nu} = 0. \tag{17}$$

In this way, the field equations now are given by

$$\Box\bar{h}_{\mu\nu} = 0 \tag{18}$$

$$\Box h_\varepsilon = E^2 h_\varepsilon \tag{19}$$

Solutions of eqs. (18) and (19) are plan waves [12, 13, 30]:

$$\bar{h}_{\mu\nu} = A_{\mu\nu}(\overrightarrow{p}) \exp(ip^\alpha x_\alpha) + c.c. \tag{20}$$

$$h_\varepsilon = a(\overrightarrow{p}) \exp(iq^\alpha x_\alpha) + c.c. \tag{21}$$

where

$$\begin{aligned} k^\alpha &\equiv (\omega, \overrightarrow{p}) \qquad & \omega = p \equiv |\overrightarrow{p}| \\ q^\alpha &\equiv (\omega_E, \overrightarrow{p}) \qquad & \omega_E = \sqrt{E^2 + p^2}. \end{aligned} \tag{22}$$

Equation (18) describes the dynamics of gravitational waves in standard general relativity [21, 22]. Equation (20) gives its solution. In parallel way, equations (19) and (21) describe, respectively, the dynamics and the solution for the new mode (see also[12, 13, 30]).

It should be emphasized that the dispersion law for the modes of the "curvature" field h_ε is not linear. Besides, the velocity of every "ordinary" wave mode $\bar{h}_{\mu\nu}$ is the speed of light c, i.e., as it arises from general relativity. Rather, the second equation in (22)) corresponds to the dispersion law for the mode h_ε. Because of this, the mass-energy field can be described like a wave-packet [12, 13, 30]. Also, the group-velocity of a wave-packet of h_ε centered in in the momentum $\overrightarrow{p}$ is

$$\overrightarrow{v_G} = \frac{\overrightarrow{p}}{\omega}, \tag{23}$$

which is exactly the velocity of a massive particle with mass-energy E and momentum $\overrightarrow{p}$.

Therefore, from the second of eqs. (22) and eq. (23) it is simple to express the group velocity as:

$$v_G = \frac{\sqrt{\omega^2 - E^2}}{\omega}. \tag{24}$$

As one expects that the wave-packet possesses a constant speed, then it has to have an energy [12, 13, 30]

$$E=\sqrt{(1-v_G^2)}\omega. \tag{25}$$

On the other hand, the analysis can remain in the Lorenz gauge with transformations of the type$\Box\epsilon_\nu=0$; this gauge gives a condition of transversal effect for the ordinary part of the field: $k^\mu A_{\mu\nu}=0$, but it does not guarantee the transversal effect of the total field $h_{\mu\nu}$. From eq. (15) it becomes

$$h_{\mu\nu}=\bar{h}_{\mu\nu}-\frac{\bar{h}}{2}\eta_{\mu\nu}+\eta_{\mu\nu}h_\varepsilon. \tag{26}$$

At this point, and considering that we are working in the massless case [12, 13, 30], this condition could be expressed as

$$\begin{array}{c} \Box\epsilon^\mu=0 \\ \\ \partial_\mu\epsilon^\mu=-\frac{\bar{h}}{2}+h_\varepsilon, \end{array} \tag{27}$$

which provides the total transversal effect of the field. However, in the actual (massive) case this is impossible. In fact, by applying the D'alembertian operator to the second of eqs.(27) and using the field equations (18) and (19) one arrives to

$$\Box\epsilon^\mu=E^2h_\varepsilon, \tag{28}$$

which is in contrast with the first of eqs. (27). In the same way it is possible to show that it does not exist any linear relation between the tensorial field $\bar{h}_{\mu\nu}$ and the "curvature" field h_ε. That is why a gauge in which $h_{\mu\nu}$ is purely spatial cannot be chosen (i.e. it cannot be given as $h_{\mu0}=0$, see eq. (26)) . But the traceless condition to the field $\bar{h}_{\mu\nu}$ can be written as: [30]

$$\begin{array}{c} \Box\epsilon^\mu=0 \\ \\ \partial_\mu\epsilon^\mu=-\frac{\bar{h}}{2}. \end{array} \tag{29}$$

These equations imply

$$\partial^\mu\bar{h}_{\mu\nu}=0. \tag{30}$$

In order to preserve the conditions $\partial_\mu\bar{h}^{\mu\nu}$ and $\bar{h}=0$ one can use transformations of the like

$$\begin{array}{c} \Box\epsilon^\mu=0 \\ \\ \partial_\mu\epsilon^\mu=0. \end{array} \tag{31}$$

By taking $\overrightarrow{p}$ in the z direction, a gauge in which only A_{11}, A_{22}, and $A_{12}=A_{21}$ are different to zero can be chosen. The condition $\bar{h}=0$ gives $A_{11}=-A_{22}$. Now, substituting these equations in eq. (26), one obtains

$$h_{\mu\nu}(t,z) = A^{+}(t-z)e^{(+)}_{\mu\nu} + A^{\times}(t-z)e^{(\times)}_{\mu\nu} + h_{\varepsilon}(t - v_G z)\eta_{\mu\nu}. \tag{32}$$

The term $A^{+}(t-z)e^{(+)}_{\mu\nu} + A^{\times}(t-z)e^{(\times)}_{\mu\nu}$ describes the two standard polarizations of gravitational waves which arise from General Relativity, while the term $h_{\varepsilon}(t - v_G z)\eta_{\mu\nu}$ is the massive field arising from the high order theory. In other words, the function R^{ε} of the Ricci scalar generates a third polarization state for gravitational waves which is not present in standard general relativity. This third polarization has a "curvature" energy E.

Now, a simple model of Universe will be proposed, in which the dynamic of the matter is described by the Vlasov equation [17, 18, 19, 20]. However, the gravitational forces between the particles, viz., the galaxies, is now supposed to be mediated by the third mode of eq. (32) after making the assumption that at cosmological scales such a mode becomes dominant (i.e. $A^{+}, A^{-} \ll h_{\varepsilon}$) [28]. In this way the "curvature" energy E can be identified as the Dark Energy of the Universe $\simeq 10^{-29} g/cm^3$ [28, 29]. These two assumptions are exacltly the ones that concern the model of oscillating Universe in [28].

The model that we are going to discuss in this letter is parallel to the one introduced by Norstrom in [19]. All the results will be obtained adapting the ideas introduced in [17, 18, 19, 20].

In the hypothesis $A^{+}, A^{-} \ll h_{\varepsilon}$, the line element of our model will be the conformally flat one

$$ds^2 = [1 + h_{\varepsilon}(t,z)](dt^2 - dz^2 - dx^2 - dy^2). \tag{33}$$

The possible using of a similar conformally flat line element in a cosmological framework has been discussed in [28, 31].

To satisfy the condition demanding that the particles make up an ensemble with no collisions in the spacetime, the particle density must be a solution of the Vlasov equation

$$\partial_t f + \frac{p^a}{p^0}\partial_{x^a} f - \Gamma^a_{\mu\nu}\frac{p^\mu p^\nu}{p^0}\partial_{p^a} f = 0. \tag{34}$$

Here $\Gamma^{\alpha}_{\mu\nu}$ represent the usual connections, f is the particle density and p^0 is determined by $p^a (a = 1, 2, 3)$ according to the relation

$$g_{\mu\nu} p^\mu p^\nu = -1 \tag{35}$$

[17, 18, 19, 20], which expresses the condition that the four momentum p^μ lies on the mass shell of the metric (greek indices run from 0 to 3) [17].

We recall that, in general, the Vlasov- Poisson system is [17, 18, 19, 20]

$$\begin{gathered} \partial_t f + v \cdot \bigtriangledown_x f - \bigtriangledown_x U \cdot \bigtriangledown_v f = 0 \\ \triangle U = 4\pi\rho \\ \rho(t,x) = \int dv f(t,x,v), \end{gathered} \tag{36}$$

where t denotes the time and x and v the position and the velocity of the galaxies. The function $U = U(t, x)$ is the average Newtonian potential generated by the galaxies. This system represents the non-relativistic kinetic model for an ensemble of particles with no collisions, which interacts through the gravitational forces that they generate collectively [17, 18, 19, 20]. Thus, one can use such a system to describe the motion of galaxies within the Universe, thought of as pointlike particles, when the relativistic effects are negligible [17, 18, 19, 20]. In this approach, the function $f(t, x, v)$ in the Vlasov- Poisson system (36) is non-negative and gives the density on phase space of the galaxies within the Universe.

The Vlasov equation (34) implies that the function f is constant on the geodesic flow of the line element (33). The connection of the line element (33) are obtained from (note: as we are working in the linearized approach, in the following computations only terms up to first order in h_ε will be considered while high-order terms will be assumed equal to zero)

$$\Gamma^{\alpha}_{\mu\nu} = \frac{1}{2}(\delta^{\alpha}_{\nu}\partial_{\mu}h_\varepsilon + \delta^{\alpha}_{\mu}\partial_{\nu}h_\varepsilon - \frac{1}{1+2h_\varepsilon}g_{\mu\nu}\partial_{\alpha}h_\varepsilon). \tag{37}$$

In this way, the Vlasov equation in the spacetime defined by the line element (33) becomes

$$\partial_t f + \frac{p^a}{p^0}\partial_{x^a} f - \frac{1}{2}[2(p^{\mu}\partial_{\mu}h_\varepsilon)\frac{p^a}{p^0} + \frac{\partial_a h_\varepsilon}{(1+2h_\varepsilon)p^0}]\partial_{p^a} f = 0. \tag{38}$$

Now, let us recall that two quantities are important for the Vlasov equation in a curve spacetime [16]. The first is the current density

$$N^{\mu} = -\int \frac{dp}{p^0}\sqrt{g}p^{\mu} f \tag{39}$$

and the second is the stress-energy tensor

$$T^{\mu\nu} = -\int \frac{dp}{p^0}\sqrt{g}p^{\mu}p^{\nu} f,. \tag{40}$$

Here g is the usual determinant of the metric tensor that in the case of the line element (33) is given by

$$g = 1 + 2h_\varepsilon. \tag{41}$$

Both of N^{μ} and $T^{\mu\nu}$ are divergence free (conservation of energy):

$$\begin{aligned} &\nabla_{\mu}N^{\mu} \\ &\nabla_{\mu}T^{\mu\nu}. \end{aligned} \tag{42}$$

The mass shell conditions (35) can be rewritten as

$$p^0 = \sqrt{(1+h_\varepsilon)^{-1} + \delta_{ab}p^a p^b}. \tag{43}$$

From the connections (37), computing the Riemann tensor, Ricci tensor and Ricci scalar, the "effective" Einstein field equations

$$G_{\mu\nu} = T_{\mu\nu}, \tag{44}$$

can be obtained together with the "effective" Klein - Gordon equation

$$\Box h_\varepsilon = -T, \tag{45}$$

where $T \equiv T^\mu_\mu$ is the trace of the stress-energy tensor.

To simplify the computations the analysis can be performed in a conformal frame. Thus, rescaling the stress-energy tensor in the form

$$T^{\mu\nu}_* = (1 + 3h_\varepsilon)T^{\mu\nu}, \tag{46}$$

one obtains

$$T_* = (1 + 2h_\varepsilon)T. \tag{47}$$

Note: in general, conformal transformations are performed by rescaling the line-element like [26, 27]

$$\tilde{g}_{\alpha\beta} = e^\Phi g_{\alpha\beta}. \tag{48}$$

Here we choose the scalar field as being

$$\Phi \equiv h_\varepsilon, \tag{49}$$

which also implies

$$e^\Phi = 1 + h_\varepsilon, \tag{50}$$

in our linearized approach.

Thus, equation (45) becomes

$$\Box h_\varepsilon = -T_*. \tag{51}$$

We note that the particle density is still defined on the mass shell of the starting line element $g_{\alpha\beta}$. In order to remove even this last connection with the starting frame we rescale the momentum as

$$p^\mu_* = (1 + \frac{h_\varepsilon}{2})p^\mu \tag{52}$$

and define the particle density in the new conformal frame as

$$f_*(t, x, p_*) = f(t, x, (1 - \frac{h_\varepsilon}{2})p_*). \tag{53}$$

Hence, we can write our adaptation of the Vlasov system in the following form

$$\Box h_\varepsilon = (1 + 2h_\varepsilon) \int \frac{dp_*}{p^0_*} f_*(t, x, p_*), \tag{54}$$

$$p^0_* = \sqrt{1 + \delta_{ab} p^a_* p^b_*}. \tag{55}$$

$$\partial_t f_* + \frac{p^a_*}{p^0_*} \partial_{x^a} f_* - \frac{1}{p^0_*}[p^\mu_* \partial_\mu h_\varepsilon p^a_* + \partial_a h_\varepsilon] \partial_{p^a_*} f_* = 0. \tag{56}$$

Because we want to obtain spherical symmetry to remain in correspondence with astronomical observations, we can postulate the homogeneity and isotropy of the Universe. In this case the line-element (33) assumes the general form

$$ds^2 = [1 + h_\varepsilon(t, r)](dt^2 - dr^2). \tag{57}$$

where r is the radial coordinate. Thus, in spherical coordinates, equations (54), (55) and (56) can be written as

$$-\frac{d^2 h_\varepsilon}{dt^2} + \frac{1}{r^2}\frac{d}{dr}(\frac{d}{dr} h_\varepsilon r^2) = (1 + 2h_\varepsilon)\mu(t, r), \tag{58}$$

$$\mu(t, r) = \int \frac{dp}{\sqrt{1 + p^2}} f(t, r, p), \tag{59}$$

$$\partial_t f + \frac{p}{\sqrt{1+p^2}} \partial_r f - [(\frac{d}{dt} h_\varepsilon + \frac{xp}{\sqrt{1+p^2}} \frac{1}{r}\frac{d}{dr} h_\varepsilon) p + \frac{x}{\sqrt{1+p^2}} \frac{1}{r}\frac{d}{dr} h_\varepsilon] \partial_p f = 0, \tag{60}$$

where the suffix $*$ has been removed for the sake of simplicity, and we have denoted by p the vector $p = (p_1, p_2, p_3)$ with $p^2 = |p|^2$, and also defined x for the vector $x_i = (x_1, x_2, x_3)$.

Then, wanting stationary states, following [28], we can call $\underline{f}$ the frequency of the "cosmological" gravitational wave (32), which has not to be confused with the particle density that we have previous denoted with f, and assume that $\underline{f} \ll H_0$ where H_0 is the Hubble constant (i.e. the gravitational wave is "frozen" with respect the cosmological observations). In this case it is $\frac{d}{dt} h_\varepsilon = 0$.

Thus, the system of equations which defines the stationary solutions of eqs.(58), (59) and (60), for our model of Universe is

$$\frac{1}{r^2}\frac{d}{dr}(\frac{d}{dr} h_\varepsilon r^2) = (1 + 2h_\varepsilon)\mu(r), \tag{61}$$

$$\mu(r) = \int \frac{dp}{\sqrt{1 + p^2}} f(r, p), \tag{62}$$

$$p\partial_r f - \frac{1}{r}\frac{d}{dr} h_\varepsilon [(pr)p + r] \partial_p f = 0, \tag{63}$$

Conclusions

It has been shown that a weak modification of general relativity, in the linearized approach produces a spherically symmetric and stationary model of the universe. This is due to the presence of a third mode of polarization in the linearized gravity in which a "curvature" energy is present. Such an energy can be, in principle, identified as the Dark Energy. The model can also help to a better understanding of the framework of the Einstein-Vlasov system.

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